
GEOMETRY – TRIANGLES AND ANGLES

❑ INTRODUCTION

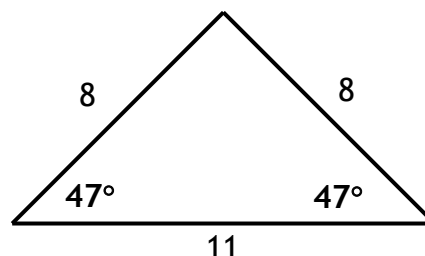
An *isosceles triangle* has at least two equal sides. It's also the case that an isosceles triangle must have at least two equal angles in it, namely the angles that are opposite the equal sides.

It's a fact from geometry that the sum of the three inside angles of any triangle is always 180° . For example, in the isosceles triangle above, the top angle must be 86° , because

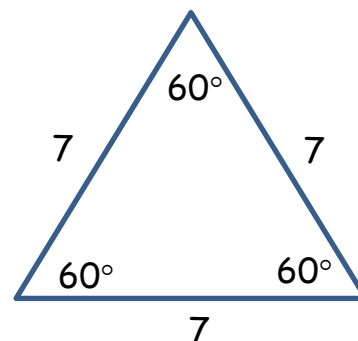
$$47^\circ + 47^\circ + 86^\circ = 180^\circ$$

Because an *equilateral triangle* has three equal sides, all the angles must be the same — namely 60° each — since

$$60^\circ + 60^\circ + 60^\circ = 180^\circ$$



An Isosceles Triangle



An Equilateral Triangle
(It's also isosceles)

Homework

1. Two of the angles in a triangle are 100° and 30° . What is the measure of the third angle?
2. Two of the angles in an isosceles triangle are 35° and 35° . Find the measure of the third angle.

3. Name the angles in any equilateral triangle.
4. True/False:
 - a. The two acute angles (the ones that are not 90°) in a right triangle must be the same.
 - b. The two acute angles in a right triangle have a sum of 90° .

□ **ANGLE PROBLEMS**

Note that the following problems deal with the three **angles** in a triangle, not their sides. Notice that the problems do not specify what the sum of the three angles is — that's because we know that **the sum of the inside angles in any triangle is 180°** (and it's your job to remember that fact!).

EXAMPLE 1: **The second angle of a triangle is 12° less than 4 times the first angle. The third angle is 30° less than the second angle. What is the measure of each angle?**

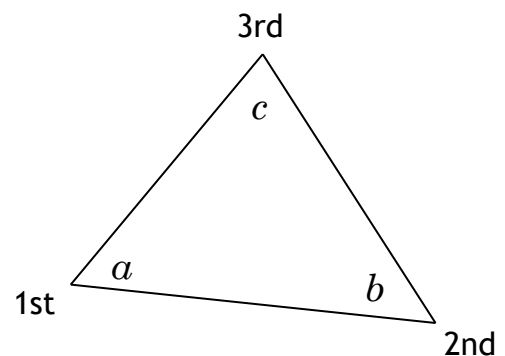
Solution: First, since the problem talks about the three angles of a triangle, the key to the equation will be that the angles must total 180° . We start by giving names to the three angles inside the triangle:

a = 1st angle

b = 2nd angle

c = 3rd angle

Now we parse the given problem phrase by phrase:



The second angle of a triangle is 12° less than 4 times the first angle translates to

$$b = 4a - 12$$

The third angle is 30° less than the second angle becomes

$$c = b - 30$$

Our equation is

$$\begin{aligned}
 a + b + c &= 180 && \text{(angles total } 180^\circ\text{)} \\
 \Rightarrow a + \boxed{4a - 12} + \boxed{b - 30} &= 180 && \text{(substitute for } b \text{ \& } c\text{)} \\
 \Rightarrow 5a - 42 + b &= 180 && \text{(combine like terms)} \\
 \Rightarrow 5a - 42 + \boxed{4a - 12} &= 180 && \text{(substitute for } b\text{)} \\
 \Rightarrow 9a - 54 &= 180 && \text{(combine like terms)} \\
 \Rightarrow 9a &= 234 && \text{(add 54 to each side)} \\
 \Rightarrow \mathbf{a = 26} &&& \text{(divide by 9)}
 \end{aligned}$$

We have our final answers:

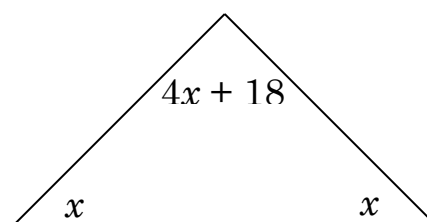
The first angle is $a = \boxed{26^\circ}$

The second angle $= b = 4a - 12 = 4(\mathbf{26}) - 12 = \boxed{92^\circ}$

The third angle $= c = b - 30 = \mathbf{92} - 30 = \boxed{62^\circ}$

EXAMPLE 2: The third angle of an isosceles triangle is 18° more than 4 times one of the equal angles.
What is the measure of each angle?

Solution: For variety, let's try solving this problem using just one variable.
Since two of the angles must be equal,



let's call each of them x . So the third angle, which is 18° more than 4 times x , can be written $4x + 18$. The sum of the three angles must be 180° :

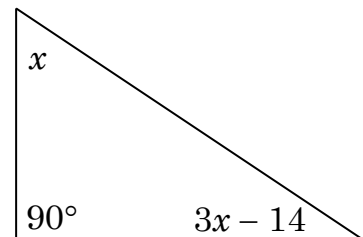
$$\begin{aligned}
 x + x + (4x + 18) &= 180 && \text{(3 angles total } 180^\circ\text{)} \\
 \Rightarrow x + x + 4x + 18 &= 180 && \text{(remove parentheses)} \\
 \Rightarrow 6x + 18 &= 180 && \text{(combine like terms)} \\
 \Rightarrow 6x &= 162 && \text{(subtract 18 from each side)} \\
 \Rightarrow \mathbf{x} &= \mathbf{27} && \text{(divide each side by 6)}
 \end{aligned}$$

$$\text{and so } 4x + 18 = 4(\mathbf{27}) + 18 = 126$$

Hence, the three angles are $27^\circ, 27^\circ$, and 126°

EXAMPLE 3: One of the acute angles of a right triangle is 14° less than 3 times the other acute angle. Find the measure of the larger acute angle.

Solution: Again, let's use just one variable to solve this problem. Since one of the angles in a right triangle is 90° , we need only concern ourselves with the two acute angles. Calling one of them x means that the other one must be called $3x - 14$. Since the sum of the three angles must be 180° , we get the equation

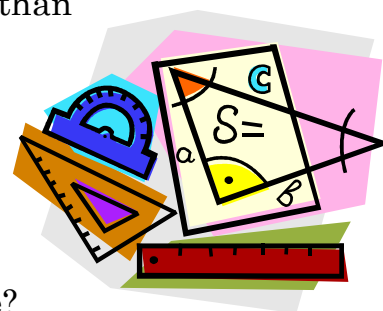


$$\begin{aligned}
 x + (3x - 14) + 90 &= 180 && \text{(3 angles total } 180^\circ\text{)} \\
 \Rightarrow x + 3x - 14 + 90 &= 180 && \text{(drop the parentheses)} \\
 \Rightarrow 4x + 76 &= 180 && \text{(combine like terms)} \\
 \Rightarrow 4x &= 104 && \text{(subtract 76 from each side)} \\
 \Rightarrow \mathbf{x} &= \mathbf{26} && \text{(divide each side by 4)} \\
 \Rightarrow 3x - 14 &= 3(\mathbf{26}) - 14 = 64
 \end{aligned}$$

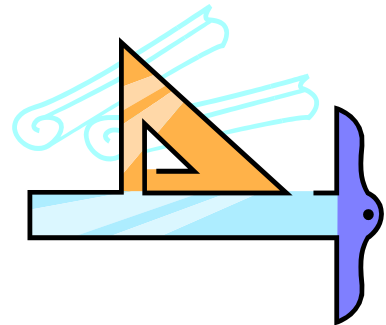
Therefore, the larger acute angle is 64°

Homework

5. One of the acute angles of a right triangle is 18° more than 2 times the other acute angle. Find the measures of the two acute angles.
6. The third angle of an isosceles triangle is 15° more than 3 times one of the equal angles. What is the measure of each angle?
7. The second angle of a triangle is 16° more than 3 times the first angle. The third angle is 125° less than the second angle. What is the measure of each angle?
8. The third angle of an isosceles triangle is 4° less than 2 times one of the equal angles. What is the measure of each angle?
9. The second angle of a triangle is 38° less than 4 times the first angle. The third angle is 13° more than the second angle. What is the measure of each angle?
10. One of the acute angles of a right triangle is 6° less than 5 times the other acute angle. Find the measures of the two acute angles.
11. One of the acute angles of a right triangle is 10° less than 3 times the other acute angle. Find the measures of the two acute angles.
12. The third angle of an isosceles triangle is 8° less than 2 times one of the equal angles. What is the measure of each angle?
13. The second angle of a given triangle is 66° more than its first angle, while its third angle is 18° less than 4 times the first angle. Find the measure of each angle.
14. One of the acute angles of a right triangle is 18° less than 2 times the other acute angle. Find the measures of the two acute angles.



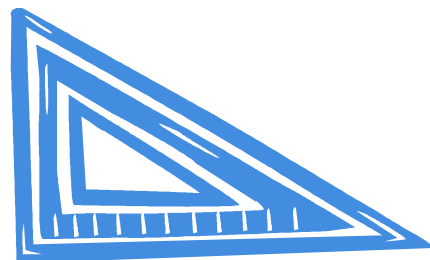
15. One of the acute angles of a right triangle is 60° less than 2 times the other acute angle. Find the measures of the two acute angles.
16. The third angle of an isosceles triangle is 68° less than 2 times one of the equal angles. What is the measure of each angle?
17. The second angle of a triangle is 20° less than the first angle. The third angle is 20° more than twice the second angle. What is the measure of each angle?



Review Problems

18. The second angle of a triangle is 30° more than the first angle, while the first angle is 26° more than 3 times the third angle. What is the measure of each angle?
19. One of the acute angles of a right triangle is 18° more than 5 times the other acute angle. What is the measure of each acute angle?
20. The first angle of a triangle is 1° more than 8 times the third angle, while the second angle is 47° more than 3 times the third angle. What is the measure of each angle?
21. The third angle of a triangle is 90° less than 10 times the second angle, while the first angle is 50° less than 9 times the second angle. What is the measure of each angle?
22. One of the acute angles of a right triangle is 10° less than the other acute angle. What is the measure of each acute angle?
23. The second angle of a triangle is 96° less than 3 times the third angle, while the first angle is 25° less than 3 times the third angle. What is the measure of each angle?

24. The third angle of a triangle is 38° more than 3 times the second angle, while the first angle is 43° less than the third angle. What is the measure of each angle?
25. The second angle of a triangle is 11° more than 2 times the third angle, while the first angle is 55° less than 5 times the third angle. What is the measure of each angle?
26. One of the acute angles of a right triangle is 14° less than the other acute angle. What is the measure of each acute angle?
27. The second angle of a triangle is 3° less than 4 times the first angle, while the third angle is 42° more than the second angle. What is the measure of each angle?
28. The second angle of a triangle is 32° less than 6 times the first angle, while the third angle is 42° less than the second angle. What is the measure of each angle?
29. The third angle of an isosceles triangle is 12° more than 5 times one of the equal angles. What is the measure of each angle?
30. The second angle of a triangle is 28° more than 5 times the first angle, while the third angle is 42° more than 4 times the first angle. What is the measure of each angle?
31. The second angle of a triangle is 75° more than 5 times the first angle, while the third angle is 21° less than 3 times the first angle. What is the measure of each angle?
32. The third angle of an isosceles triangle is 4° more than 20 times one of the equal angles. What is the measure of each angle?
33. The second angle of a triangle is 29° more than the first angle, while the first angle is 15° more than 5 times the third angle. What is the measure of each angle?



Solutions

- | | | |
|-------------------------------------|-------------------------------------|-------------------------------------|
| 1. 50° | 2. 110° | 3. $60^\circ, 60^\circ, 60^\circ$ |
| 4. a. F b. T | 5. $24^\circ, 66^\circ$ | 6. $33^\circ, 33^\circ, 114^\circ$ |
| 7. $39^\circ, 133^\circ, 8^\circ$ | 8. $46^\circ, 46^\circ, 88^\circ$ | 9. $27^\circ, 70^\circ, 83^\circ$ |
| 10. $16^\circ, 74^\circ$ | 11. $25^\circ, 65^\circ$ | 12. $47^\circ, 47^\circ, 86^\circ$ |
| 13. $22^\circ, 70^\circ, 88^\circ$ | 14. $36^\circ, 54^\circ$ | 15. $40^\circ, 50^\circ$ |
| 16. $56^\circ, 62^\circ, 62^\circ$ | 17. $35^\circ, 55^\circ, 90^\circ$ | 18. $14^\circ, 68^\circ, 98^\circ$ |
| 19. $12^\circ, 78^\circ$ | 20. $11^\circ, 89^\circ, 80^\circ$ | 21. $16^\circ, 70^\circ, 94^\circ$ |
| 22. $50^\circ, 40^\circ$ | 23. $43^\circ, 33^\circ, 104^\circ$ | 24. $21^\circ, 101^\circ, 58^\circ$ |
| 25. $28^\circ, 67^\circ, 85^\circ$ | 26. $52^\circ, 38^\circ$ | 27. $16^\circ, 61^\circ, 103^\circ$ |
| 28. $22^\circ, 100^\circ, 58^\circ$ | 29. $24^\circ, 24^\circ, 132^\circ$ | 30. $11^\circ, 83^\circ, 86^\circ$ |
| 31. $14^\circ, 145^\circ, 21^\circ$ | 32. $8^\circ, 8^\circ, 164^\circ$ | 33. $11^\circ, 70^\circ, 99^\circ$ |

“The beginning of knowledge
is the discovery of something
we do not understand.”

Frank Herbert